Algebra III

Mid-Term Examination

Instructions: All rings are assumed to be commutative with unity. All questions carry equal marks.

1. Prove that a ring is field if and only if it has exactly two ideals.

2. Let I be an ideal in a ring R. Prove that the set of ideals in the ring R/I is in one-one , onto correspondence with the set of ideals in R that contain I.

3. Define *characteristic* of a ring R. Prove that if $a^2 = a$ for every $a \in R$, then R has characteristic 2. Is the converse of this statement true? Justify your answer.

4. Let $\mathbb{Z}$ and $\mathbb{Q}$ denote the ring of integers and the field of rationals respectively. Prove that there is no surjective ring homomorphism from $\mathbb{Z}[X]$ to $\mathbb{Q}$.

5. Prove that a ring R is a domain if and only if any $f \in R[X]$ has at most $\deg(f)$ roots in R.

6. Let F be a field. Prove that the number of maximal ideals in the polynomial ring $F[X]$ is infinite.